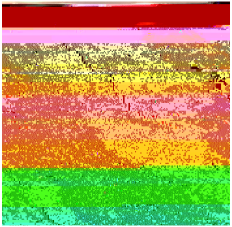

I'm Brad Rodgers, an Associate Professor here in the Department of Math and Stats at Queen's University.

My research is largely in analytic number theory and random matrix and point process theory. Problems I am interested in often involve applying ideas from functional and harmonic analysis and probability to number theory. Two examples of topics I have been interested in recently are:

The distribution of arithmetic functions in short intervals In a first course in number theory one often studies the average value of arithmetic functions or sequences on *deterministic long intervals*; for instance the prime number theorem is the statement that the likelihood a random number n in between 1 and X is prime is roughly $1/\log X$.

What can be said about the number of primes in *random short interval* $[n, n + X^{1/4}]$ where n is again chosen uniformly from 1 to X? Such a question is more subtle than the prime number theorem – this question in particular is closely connected to the fine-scale distribution of zeros of the Riemann zeta-function. Similar questions for sums of divisor functions over random short intervals are related to moments of the Riemann zeta-function. In recent work collaborators and I have answered an old question of this sort about the number of squarefree integers in random short intervals. The problem ends up being related to questions of the following sort: count the number of reduced rational solutions to

$$a_1/q_1 + a_2/q_2 + a_3/q_3 + a_4/q_4 = 0$$

where $|q_i| \le Q$ and $|a_i/q_i| \le Q^{-1/10}$. Are most solutions given by the obvious 'paired' solutions, in which e.g. $a_1/q_1 = -a_2/q_2$ and $a_3/q_3 = -a_4/q_4$?

The distribution of random or special trigonometric polynomials An old theorem Salem and Zygmund says that for a "typical" choice of independent random coefficients $\epsilon_1 = \pm 1$, $\epsilon_2 = \pm 1$, ...

$$\sup_\theta \left| \sum_{n=1}^{N} \epsilon_n e^{in\theta} \right| \quad \sqrt{N \log N};$$

as N grows. What happens when the coefficients ϵ_n are chosen in a more structured way? For instance I would like to know if the same is true when ϵ_n is a random multiplicative function. One can also ask extremal questions. It is known that there exists choices of ϵ_n such that the above quantities are $O(\sqrt{N})$. (Parseval tells us this is the smallest one could hope for.) Can anything sensible be said when the coefficients ϵ_n are restricted to be a multiplicative function?

If you find topics of this sort interesting or would like to hear more about my research, do not hesitate to contact me at brad.rodgers@queensu.ca