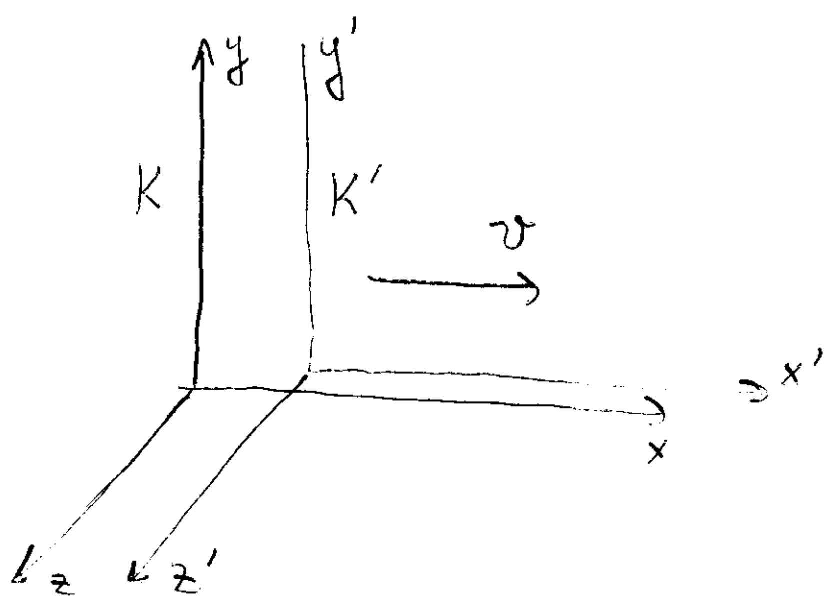
Special Relativity

Relativity theory of spacetime + connection with laws of physics

Spacetime collection of all events



Prior to 1905 - Galilean transformation

$x' = x - vt$ $\qquad$ $\vec{v}$ rel. vel. of frames $= v\hat{x}$ constant

$y' = y \quad z' = z \quad t' = t$

$\vec{u}$ = vel. of particle as measured in K

$\vec{u}'$ = vel of particle as measured in K'

$= \dfrac{d\vec{x}'}{dt'} \Rightarrow u'_x = u_x - v$

$u'_y = u_y \quad u'_z = u_z$

$x' = 0 \Rightarrow x = vt$

transformations are linear and invertible

velocity addition laws

Newton's laws $\quad \vec{F} = m\vec{a}$

$$\vec{a} = \frac{d\vec{u}}{dt}$$

$$\vec{a}' = \frac{d\vec{u}'}{dt'} = \frac{d\vec{u}}{dt} \qquad \vec{v} = \text{const.}$$

$$t' = t$$

+ $\vec{F}' = \vec{F}$ so $\vec{F}' = m\vec{a}'$

Maxwell's equations in vacuum

$$\vec{\nabla} \times \vec{B} = -\frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = -\nabla^2 \vec{B} + \cancel{\vec{\nabla}(\vec{\nabla} \cdot \vec{B})}^{\,0}$$

$$= -\frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla} \times B$$

$$= -\frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} \qquad \boxed{\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{B} = 0}$$

Now $$\frac{\partial}{\partial t} = \frac{\partial t'}{\partial t}\frac{\partial}{\partial t'} + \frac{\partial x'}{\partial t}\frac{\partial}{\partial x'} = \frac{\partial}{\partial t'} - v\frac{\partial}{\partial x'}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x'}$$

so under [illegible]

$$\left(-\frac{\hbar^2}{2m}\nabla'^2 - i\hbar\frac{\partial}{\partial t'} + i\hbar v\frac{\partial}{\partial x'}\right)\psi = V\psi$$

$$\psi = \psi' e^{i\frac{m}{\hbar}vx' + i\frac{mv^2}{2\hbar}t'}$$

gives $$\left(-\frac{\hbar^2}{2m}\nabla'^2 - i\hbar\frac{\partial}{\partial t'}\right)\psi' = V'\psi'$$

with $V = V'$

Back to Maxwell's equations

One passibility is that light propogates in a medium - the ether. (Imagine we were writing equations for sound ...)

Einstein's 2nd postulate

Speed of light is the same in all frames of reference [illegible]

Consider a spherical light signal at $t = 0$

Wave front will be at t, x, y, z so that

$$c^2 t^2 = x^2 + y^2 + z^2$$ Einstein's 2nd postulate

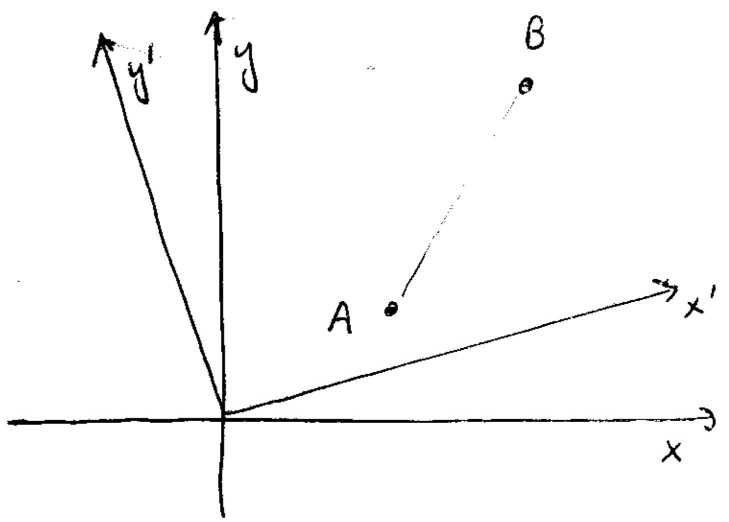
Could use K' coordinates

$$c^2 t'^2 = x'^2 + y'^2 + z'^2$$

$$\Rightarrow \quad c^2 t^2 - x^2 - y^2 - z^2 = c^2 t'^2 - x'^2 - y'^2 - z'^2$$

Let t_A, x_A, y_A, z_A $\quad t_B, x_B, y_B, z_B$

Reminiscent of rotations in plane



$$s^2 = (x_A - x_B)^2 + (y_A - y_B)^2$$

$$= (x_A' - x_B')^2 + (y_A' - y_B')^2$$

where

$$x' = \cos\theta\, x + \sin\theta\, y$$
$$y' = -\sin\theta\, x + \cos\theta\, y$$

for events, the "interval" $c^2(t_A - t_B)^2 - (\vec{x}_A - \vec{x}_B)^2$

is a spacetime invariant

$$ct' = \cosh \psi\, ct - \sinh \psi\, x$$

$$x' = -\sinh \psi\, ct + \cosh \psi\, x$$

+ use $\cosh^2 \psi - \sinh^2 \psi = 1$ identity!

Note $x' = 0 \Rightarrow x = vt$

$$x' = 0 \quad \Rightarrow \quad x = \frac{\sinh \psi}{\cosh \psi} ct \qquad \beta = \frac{v}{c} = \tanh \psi$$

$$\cosh \psi = \frac{1}{(1 - \tanh^2 \psi)^{1/2}} = \frac{1}{(1-\beta^2)^{1/2}} = \gamma$$

$$ct' = \gamma\left(ct - \frac{v}{c} x\right) \qquad x' = \gamma\left(x - \frac{v}{c} ct\right)$$

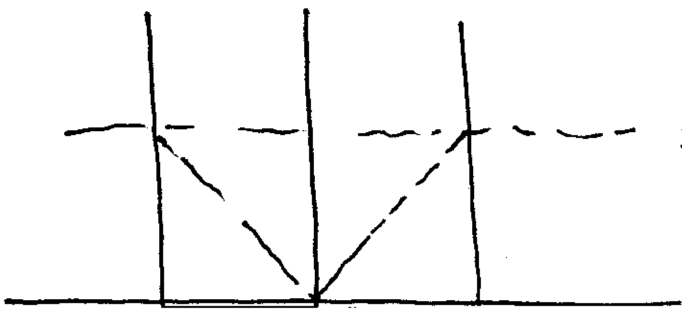
4-vector notation

$$x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z)$$

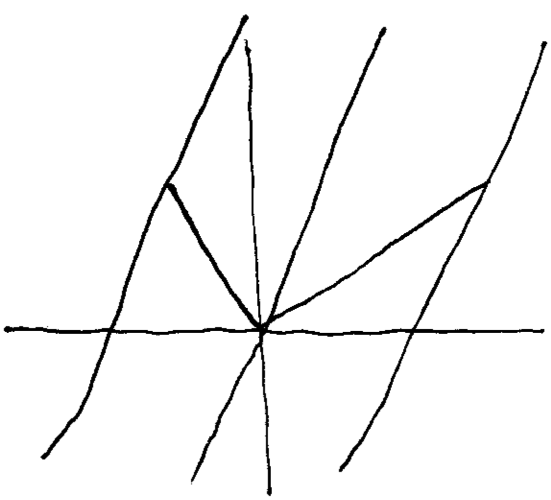
$$x^{1\prime} = \gamma(x^1 - \beta x^0)$$


ct
ct'

Einstein's train paradox



signal received at same time in rest frame of train



Galilean

signals still received at same time. Light to left & right moves at different speeds

Special Relativity

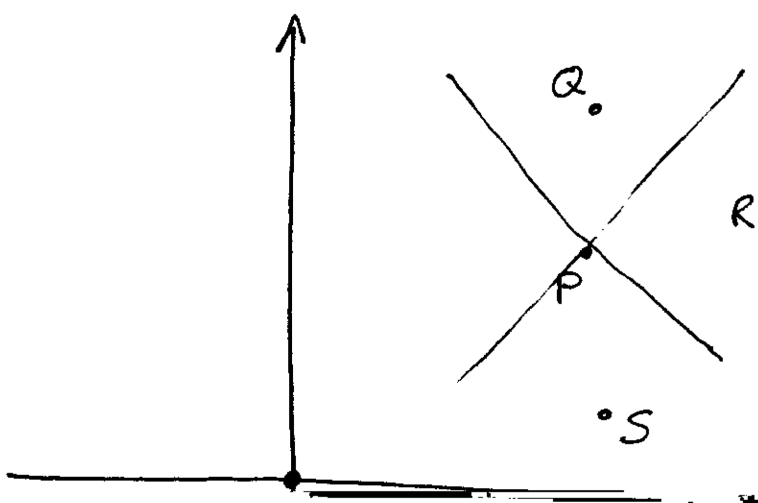
light moves at speed c

General 4-vector

$$A^\mu = (A^0, A^1, A^2, A^3)$$
$$= (A^0, \vec{A})$$

$$A^{0\prime} = \gamma(A^0 - \beta A^1) \qquad A^{2\prime} = A^2$$
$$A^{1\prime} = \gamma(A^1 - \beta A^0) \qquad A^{3\prime} = A^3$$

$$(A^{0\prime})^2 - \vec{A}^{\prime}\cdot\vec{A}^{\prime} = (A^0)^2 - \vec{A}\cdot\vec{A}$$ Lorentz invariant



for event P, spacetime is divided into
future, past, spacelike separated
Q S R

$$(\Delta s)^2 = (x_{P,0} - x_{Q,0})^2 - |\vec{x}_P - \vec{x}_Q|^2$$

spacetime invariant

> 0 if timelike separated (past or future)

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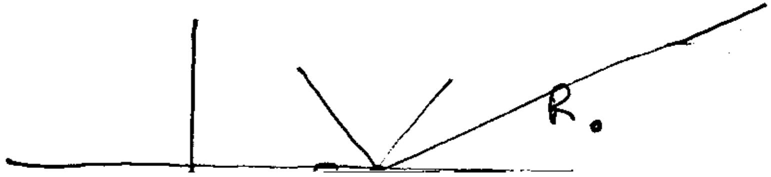
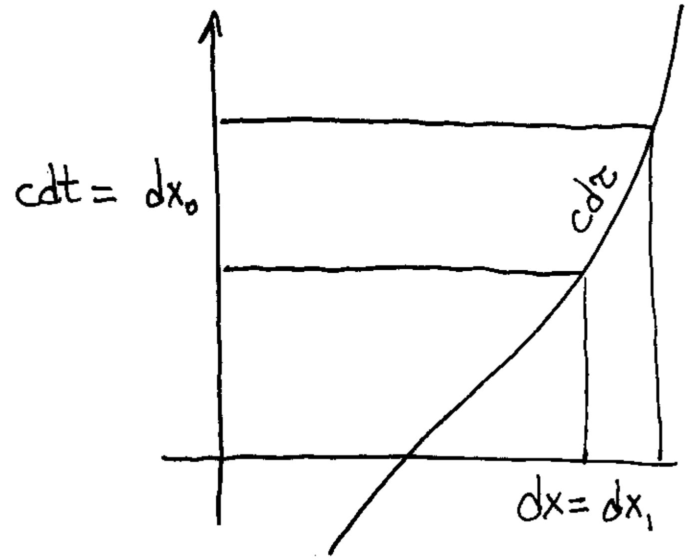
$t_P' > t_R'$





$$c^2 d\tau^2 = c^2 dt^2 - dx^2$$ Lorentz invariant

$$= c^2 dt^2 (1 - \beta^2)$$

$$d\tau = \frac{dt}{\gamma}$$ time dilation

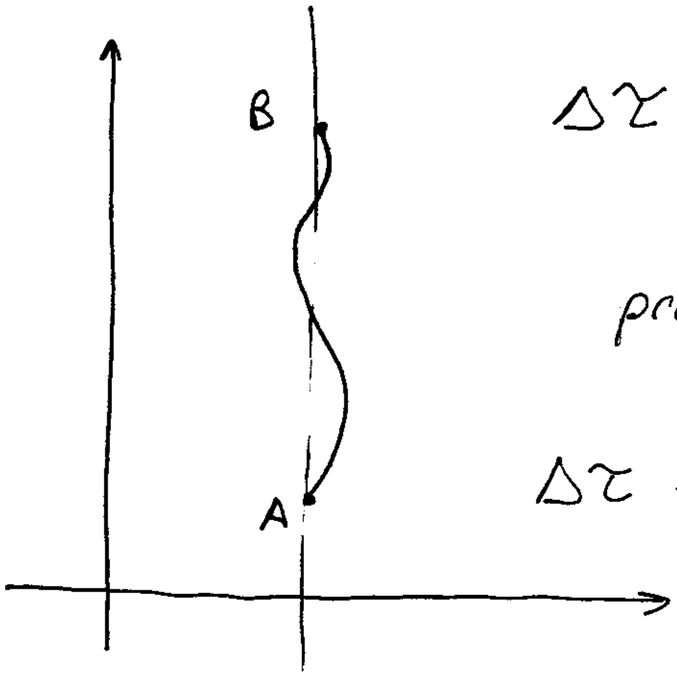
$d\tau$ = proper time

time measured by clock carried by particle

dt time interval measured by two clocks at different places in lab

Consider two timelike seperated events.

Find worldline of particle in constant motion
Let this particle define rest frame.



$$\Delta\tau = \int_A^B dt = t_B - t_A$$

proper time for this inertial observer

$\Delta\tau$ for another observer

$$= \int_A^B dt\sqrt{1-\beta^2} \leq \Delta\tau_{inertial}$$

Stay-at-home twin is older!

Can't we treat accelerating twin as twin at rest?

Yes, but requires more general treatment of spacetime.