Experiment 1. He pulled a loop of wire to the right through a

Electrodynamics

$$\mathcal{E} = \oint_C \vec{E}' \cdot d\vec{l}$$

integral of $\vec{E}'$ along $d\vec{l}$

$\vec{E}'$ is field in rest frame of $d\vec{l}$

Faraday finds $\mathcal{E} = -k \dfrac{dF}{dt}$

sign: Lenz's law - induced mag. field opposes changing flux.

k - set by units.

$$\oint_C \vec{E}' \cdot d\vec{l} = -k \frac{d}{dt} \int_S \vec{B} \cdot \hat{n} \, da$$

NB. C need not correspond to physical wire!

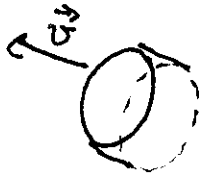
total time derivative

F changes if C changes or if $\vec{B}$ itself changes

A statement about fields!

Lets suppose C moves with velocity $\vec{v}$ (assume constant)

$\vec{B}$ may have explicit time-dependence



or $\vec{B}$ may change with position, + hence $\vec{B}$ at surface changes with time.

$$\frac{d\vec{B}}{dt} = \frac{\partial \vec{B}}{\partial t} + \left(\frac{d\vec{x}}{dt}\cdot\vec{\nabla}\right)\vec{B}$$

$$= \frac{\partial \vec{B}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{B}$$

$$= \frac{\partial \vec{B}}{\partial t} + \vec{\nabla}\times\left(\vec{v}\times\vec{B}\right) + \vec{v}\,\cancelto{0}{(\vec{\nabla}\cdot\vec{B})}$$

$$\frac{d}{dt}\int_S (\vec{B}\cdot\hat{n})\,da = \int_S \frac{\partial\vec{B}}{\partial t}\cdot\hat{n}\,da + \int \vec{\nabla}\times(\vec{v}\times\vec{B})\cdot\hat{n}\,da$$

$$= \int_S \frac{\partial\vec{B}}{\partial t}\cdot\hat{n}\,da + \oint_C (\vec{v}\times\vec{B})\cdot d\vec{l}$$

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Consider a charges particle at rest in moving circuit

As viewed in rest frame of moving circuit, it experiences an electric force $\vec{F} = q\vec{E}'$

As viewed in lab, it experiences both electric & magnetic forces $\vec{F} = q\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)$

Hence, since $\vec{F}$'s should be equal (in Galilean relativity)

$$\boxed{k = \frac{1}{c}} \qquad \vec{E}' = \vec{E} + \frac{\vec{v}}{c} \times \vec{B}$$

for fixed circuit
$$\oint_C \vec{E} \cdot d\vec{l} = \int_S (\vec{\nabla} \times \vec{E}) \cdot \hat{n}\, da$$

$$= -\frac{1}{c} \int_S \frac{\partial \vec{B}}{\partial t} \cdot \hat{n}\, da$$

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{E} + \frac{1}{c} \frac{\partial \vec{B}}{\partial t} = 0}$$

Notes: Faraday's law is correct in SR

Transformation law is only valid in $\frac{v}{c} \ll 1$ limit

(i.e., correct to $O(v^2/c^2)$)

Maxwell's displacement current

In general $\quad \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$

$$\frac{\partial}{\partial t}\left(\vec{\nabla} \cdot \vec{E} = 4\pi \rho \right) \Rightarrow \frac{\partial \rho}{\partial t} = \frac{1}{4\pi} \vec{\nabla} \cdot \frac{\partial \vec{E}}{\partial t}$$

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[illegible]'s theorem

$$\vec{E}\cdot\left(\vec{\nabla}\times\vec{B}\right) = -\vec{\nabla}\cdot\left(\vec{E}\times\vec{B}\right) + \vec{B}\cdot\left(\vec{\nabla}\times\vec{E}\right)$$

$$= -\vec{\nabla}\cdot\left(\vec{E}\times\vec{B}\right) - \frac{1}{c}\vec{B}\cdot\frac{\partial\vec{B}}{\partial t}$$


Amperian loop

Surface 1 $\qquad \vec{E} = 0 \qquad \vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{J}$

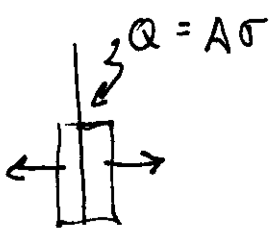
Get B_ϕ from Amperian loop

$$\int da\, \hat{n} \cdot \vec{\nabla} \times \vec{B} = \oint d\vec{l} \cdot \vec{B} = 2\pi s B_\phi$$

$$= \frac{4\pi}{c} \int da\, \hat{n} \cdot \vec{J} = \frac{4\pi}{c} I \qquad B_\phi = \frac{2I}{cs}$$

Surface 2
Same loop! $\qquad \vec{J} = 0 \quad$ but $\quad \frac{\partial \vec{E}}{\partial t} = 0$

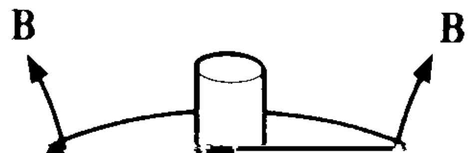
single plate



$$2E \times A = 4\pi\sigma A \qquad E = 2\pi\sigma$$

Disk is kept at fixed Ω

Begin with a small current which gives rise to B-flux thru disk. What happens?



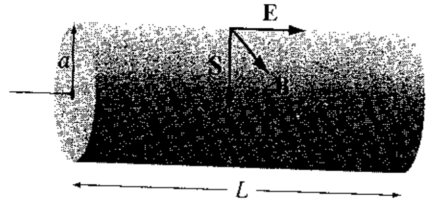


Figure 8.1

Imagine a very long solenoid with radius R, n turns per unit length, and current I. Coaxial with the solenoid are two long cylindrical

$$\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{B}$$

energy flux $\frac{\text{energy}}{\text{time area}}$

$[\vec{S}] = [\text{energy den.} \times c]$

$$\vec{\wp} = \text{momentum density}$$

$$= \frac{1}{4\pi c} \vec{E} \times \vec{B}$$

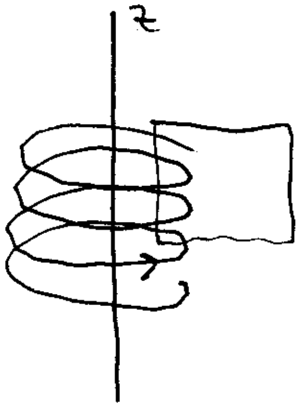
but $p = E/c$
for photons

Electric field: radial $\quad \vec{\nabla}\cdot\vec{E} = 4\pi\rho$

or $\quad 2\pi l s E_s = 4\pi Q \quad \Rightarrow \quad E_s = \dfrac{2Q}{ls}$

for $a < s < b$

$\vec{B}$ along symmetry (z) axis



B is zero outside

$$B_z \cdot l = \frac{4\pi}{c} I_{encl}$$

$$= \frac{4\pi}{c} n I l$$

so $\quad B_z = \begin{cases} \dfrac{4\pi}{c} n I & s < R \\ 0 & s \geq R \end{cases}$

in region $\quad a < s < R \quad$ we have $\quad \mathcal{G}_\phi = \dfrac{1}{4\pi c}(\vec{E}\times\vec{B})\cdot\hat{\phi}$

$$= -\frac{1}{4\pi}\frac{2Q}{ls}\frac{4\pi}{c^2} n I$$

$$= -2QnI$$

Turn off magnetic field and induced $\vec{E}$ in ϕ-direction causes cylinders to rotate

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\int da\, \hat{n} \cdot (\vec{\nabla} \times \vec{E}) = \oint \vec{E} \cdot d\vec{l}$$

$$= (E_\phi \cdot 2\pi a \qquad \text{inner}$$

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$$= -\frac{1}{c}\int \frac{\partial \vec{B}}{\partial t} \cdot \hat{n}\, da$$

$$= -\frac{1}{c}\frac{d}{dt}\int \vec{B} \cdot \hat{n}\, da$$

$$= -\frac{4\pi}{c^2} n \frac{dI}{dt} \cdot \begin{cases} \pi a^2 & \text{inner} \\ \pi a^2 & \end{cases}$$

$$E_\phi = \begin{cases} -\dfrac{2\pi}{c^2} n a \dfrac{dI}{dt} & \text{inner} \\[2ex] -\dfrac{2\pi}{c^2} n \dfrac{R^2}{b} \dfrac{dI}{dt} & \text{outer} \end{cases}$$

Torque

$$N_{inner} = a \times Q \times E_\phi \hat{z} = -\frac{2\pi}{c^2} n Q a^2 \frac{dI}{dt} \hat{z}$$

$$N_{outer} = b \times (-Q) \times E_\phi \hat{z} = \frac{2\pi}{c^2} n Q R^2 \frac{dI}{dt} \hat{z}$$

$$\vec{L} = \int \left(\vec{N}_{inner} + \vec{N}_{outer} \right) dt \qquad \int \frac{dI}{dt} dt = -I$$

$$= \frac{2\pi}{c^2} n Q I \left(a^2 - R^2 \right) \hat{z}$$

$$\vec{P}_\phi = -\frac{2QnI}{lsc^2}$$

l R

What if we "turn off" electric field?

Connect cylinders by a wire

Then Lorentz force on radially flowing charge in B_z gives rise to torque on cylinder